III Semester M.Sc. Degree Examination, December 2014
(Semester Scheme) (NS)
MATHEMATICS
M – 305 : Mathematical Methods

Time : 3 Hours Max. Marks : 80

Instructions : *i) Solve **any five** questions choosing atleast **two** from each Part **A** and Part **B**.*

*ii) **All** questions carry **equal** marks.*

PART – A

1. a) Explain the method of successive approximations to solve the volterra integral equation $y(x) = f(X) + \lambda\int_a^x K(x, t)\, y(t)\, dt$. Prove that its solution converges. **6**

   b) Transform the following into integral equation $y'' - \lambda y(x) = f(x)$; $x > 0$, $y(0) = 1$, $y'(0) = 0$; where λ is constant. **5**

   c) Solve the following integral equation $Q(x) - \int_0^x (x - s)\, Q(s) ds = 0$, by successive approximation method. **5**

2. a) Transform the differential equation $y'' - K^2 y(x) + \frac{e^{-x}}{x} y(x) = 0$; $y(0) = 0$; $y(\infty) = 0$; into equivalent Fredholm integral equation. **8**

   b) Solve the integral equation $Q(x) - \lambda \int_0^{\pi} \cos(x + t)\, Q(t)\, dt = \cos 3x$ by the method of separable or degenerate Kernel. **8**

3. a) State and prove integral representation of a non-periodic function f(x) over $-\infty < x < \infty$. **6**

b) Solve the following BVP $u'' - qu = 50e^{-2x}$ $(0 < x < \infty)$ $u(0) = u_0$; $u(\infty)$ is bounded using Fourier sine transform. **5**

c) Solve the following integral equation $Q(x) = \sin x + 2\int_0^x \cos(x-u)\, Q(u)\, du$ using Laplace transform. **5**

4. a) Reduce the following integral equation $Q(x) - \lambda\int_0^\pi K(x,t)Q(t)\,dt = 0$; where $K(x,t) = \begin{cases} \cos x \sin t, & 0 \le x < t \\ \sin x \cos t, & t \le x \le \pi \end{cases}$ to an equivalent boundary value problem and find its solution Q(x). **10**

b) Solve the initial value problem $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, -\infty < x < \infty,\ t > 0$

u = f(x) when t = 0, $-\infty < x < \infty$. Find u(x, t) when $f(x) = \begin{cases} 1 & \text{for } x > 0, \\ 0 & \text{for } x < 0 \end{cases}$. **6**

PART – B

5. a) Define asymptotic expansion of a function as $x \to 0$ and as $x \to \infty$, integrate by parts to find asymptotic expansion of $I(x) = \int_x^\infty e^{-t^2} dt$ as $x \to \infty$. **4**

b) State and prove Watson's lemma. **6**

c) Find the asymptotic approximate value of the following as $x \to \infty$. **6**

i) $I(x) = \int_{-\pi/2}^{\pi/2} (t+2)e^{-x\cos t} dt$

ii) $J_x(x) = \frac{1}{\pi}\int_0^\pi \cos(x\sin t - xt)\,dt$.

6. a) Find a two term regular perturbation solution of the following :

i) $y'' + y = \in (y' - \frac{1}{3}(y')^3);\ y(0) = a;\ y'(0) = 0.$

ii) $y'' + (1 - \in x)\,y = 0;\ y(0) = 1;\ y'(0) = 0.$ **6**

b) Apply the Poincare – Lindstedt method to find two term approximate periodic solution of $u'' + u + \in u^3 = 0; u(0) = a; u'(0) = 0.$ **6**

c) Find a two term approximate solution for small $\in$ of the problem $y'' = \in (\sin x)y;$ $y(0) = 1;\ y'(0) = 1.$ **4**

7. a) Find a 1-term uniformly valid solution of the singular perturbation problem $\in y'' + y' + y = 0;\ y(0) = \alpha,\ y(1) = \beta.$ **8**

b) Apply the boundary layer theory to find a 1-term perturbation solution of $\in y'' + x^2 y' - y = 0;\ y(0) = y(1) = 1.$ **8**

8. a) Obtain the WKB 1-term approximate solution of $\in^2 y'' = Q(x)\,y$. **8**

b) Solve any two of the following non-linear differential equations **8**

i) $yy'' + a(y'^2 + 1) = 0;$

ii) $x^2 yy'' + (xy' - y)^2 - 3y^2 = 0;$ (Hint : $y'/y = u$).